Name:

Instructions: Upload a pdf of your submission to **Gradescope**. This worksheet is worth 20 points: up to 8 points will be awarded for accuracy of certain parts (to be determined after the due date) and up to 12 points will be awarded for completion of parts not graded by accuracy.

(1) Let
$$A \in \mathbb{R}^{2\times 4}$$
 be given by $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ -4 & -3 & -2 & -1 \end{pmatrix}$.

Evaluate the following matrix expressions. You may use properties of the relevant matrix operations to simplify your calculations.

(a) AA^{\top}

- **(b)** $A(A^{T}A)$
- (c) $(AA^{\top})^{-1}$

(2) Let
$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
 and $B = \begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

Evaluate the following matrix expressions. You may use properties of the relevant matrix operations to simplify your calculations.

(a) *AB*

- (b) A^{-1}
- (c) B^{-1}
- (d) $(BA)^{-1}$
- (e) $(ABA)^{\top}$
- (f) A^2B^2
- (3) The following matrices in $\mathbb{R}^{2\times 2}$ are expressed using some constant in \mathbb{R} . For each matrix, determine all possible values for the constant such that said matrix is invertible.
 - (a) $\begin{pmatrix} 1 & 0 \\ 0 & a \end{pmatrix}$
 - **(b)** $\begin{pmatrix} 1 & 0 \\ b & 2 \end{pmatrix}$
 - (c) $\begin{pmatrix} c^3 + 2 & 3c^2 \\ 0 & 0 \end{pmatrix}$